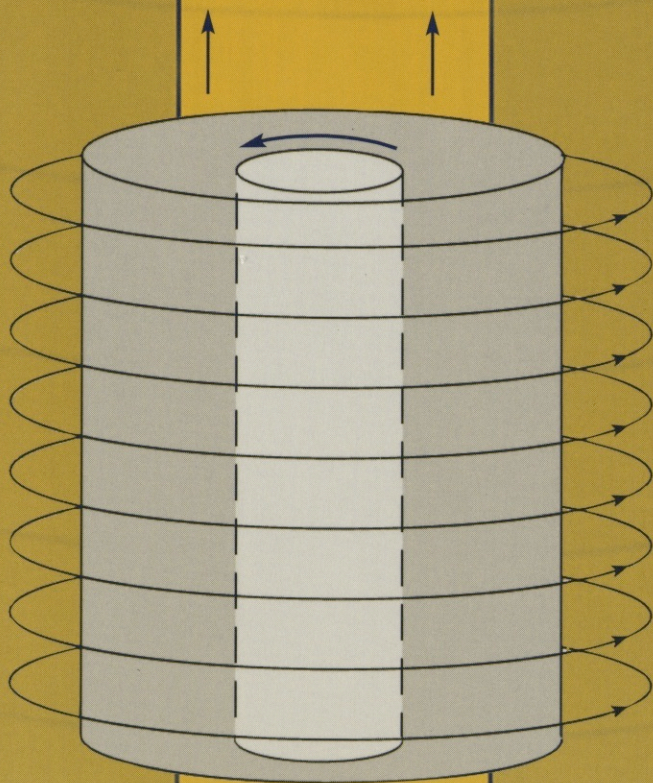


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# ELECTROMAGNETIC THEORY

ATTAY KOVETZ



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QC  
670  
.K693  
2000

**OXFORD**  
UNIVERSITY PRESS

Great Clarendon Street, Oxford OX2 6DP

Oxford University Press is a department of the University of Oxford.  
It furthers the University's objective of excellence in research, scholarship,  
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Oxford New York

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Nairobi Paris São Paulo Singapore Taipei Tokyo Toronto Warsaw

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Published in the United States  
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First published 2000

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A catalogue record for this book is available from the British Library

Library of Congress Cataloging in Publication Data  
Kovetz, Attay.

Electromagnetic theory / Attay Kovetz.

Includes bibliographical references and index.

1. Electromagnetic theory. I. Title.

QC670 .K693 2000 537.12 21-dc21 99-044818

ISBN 0 19 850604 X (Hbk acid-free paper)

ISBN 0 19 850603 1 (Pbk acid-free paper)

Typeset by  
Newgen Imaging Systems (P) Ltd., Chennai, India

Printed in Great Britain by  
Biddles Ltd, Guildford and Kings Lynn

# Preface

This book is the outcome of a course entitled ‘Analytical Electromagnetism’, which I have taught on and off during the last 20 years at Tel Aviv University. The course is attended by physics students during the latter half of their second year, or at the beginning of their third year, undergraduate studies. It is at an advanced level: the students have all been exposed to electromagnetism, as well as to special relativity, through several courses (both in the class and in the laboratory) on classical and modern physics.

Obviously, then, the students who attend such an advanced undergraduate course have all seen Maxwell’s equations in one form or another. They have all heard of Poynting’s vector, electric displacement, Joule heating, Lorentz force and electromagnetic waves. But, despite this familiarity, the average student is uncertain about the concepts and laws of electromagnetic theory. And this uncertainty stands in remarkable contrast to the confidence with which students regard the concepts and laws of mechanics. Unless there are some gaping faults in the logical structure of electromagnetic theory, the reason for these difficulties must lie in the way in which this theory is usually presented.

I believe that an advanced undergraduate course affords a good opportunity for presenting electromagnetism as a coherent and logical theory. This is the purpose of this book. Its level is roughly that of other texts on electromagnetism that start from Maxwell’s equations. In particular, it assumes that the reader requires little or no motivation for the introduction of various electromagnetic concepts and definitions. But it differs from most other textbooks in presenting electromagnetism as a *classical* theory, based on principles that are independent of the atomic constitution of matter. Whereas the concepts and principles of classical mechanics are never claimed to depend on the fact that ordinary (terrestrial) matter consists of atoms and molecules, textbooks on electromagnetism abound with discussions of stretched molecules forming tiny dipoles and of Ampèrian currents associated with atomic electrons, all dating back to Lorentz’s 90-year-old *Theory of electrons*. Not only are the students asked to believe that the average value of a quantity can be found in the absence of any knowledge regarding its distribution, they are also expected to forget that the existence of atoms can only be established on the basis of quantum mechanics, using the very Hamiltonian operators that are suggested by electromagnetism.

Obviously, then, contemporary authors, who no longer believe the classical theory of electrons—and who look to the solid state physicists for an explanation of its well-known successes—are not being quite frank with their readers. Clearly, a presentation of electromagnetic theory that avoids the atomic constitution of matter is to be preferred on both didactic and logical grounds.

The concepts of force and energy are a major source of confusion for students of electromagnetism. They know that force and energy are subject to the laws of mechanics and thermodynamics, and they had no special difficulties in mastering and applying these concepts in their studies of those two disciplines. But in electromagnetism force and energy reappear in strange and apparently conflicting forms. There is a reason for this: the discussion of force and energy requires the establishment of a definite relationship between mechanics, thermodynamics and electromagnetism. Textbooks are only too often strangely vague about this relationship. The treatment is usually wordy, in the worst thermodynamic tradition; indeed the laws of mechanics and thermodynamics are seldom stated in mathematical form. And the manner in which these laws must be modified in order to account for both electromagnetic momentum *and* energy flux remains unclear. Special attention is therefore paid in this book to the precise link between the three major disciplines of classical physics.

The outline of the book is as follows: starting from the law of charge conservation, Faraday's laws and the Maxwell–Lorentz aether relations, the principles of electromagnetism are stated, in Chapters 1–5, in tensor form. Since undergraduate students regard tensors with apprehension, there is a short chapter on antisymmetric tensors which, I hope, will provide not only a gentle introduction, but also the necessary motivation for their use. Besides taking care of questions regarding transformations and invariance, the tensorial formulation has the added advantage that students will not have to relearn electromagnetism when they take a course in general relativity. For, although curved space-time is not mentioned in these chapters, the possibility is nowhere denied.

Chapter 6 deals with the motion of charged particles. Since this subject is closely connected with measurement (the oscilloscope), it is a good place for introducing *Système International* (SI) electromagnetic units.

Chapters 7–11 cover what is roughly the subject matter of pre-Maxwellian electromagnetism—polarization and magnetization, electrostatics, slowly varying currents and their magnetic effects. But the treatment is modern: relativistic effects are not always ignored, and the student is encouraged to develop considerable skill in dealing with moving bodies. I have left out the treatment of alternating-current circuits because they are adequately treated in other courses (but there is a section on the skin effect).

Electromagnetic radiation and wave propagation are treated in Chapters 12 and 13.

Chapter 14 is a review of continuum mechanics, and introduces Coleman and Noll's method in order to establish the link between mechanics and thermodynamics

on a rational basis. Everything comes together in Chapter 15, where the complete set of laws of electromagnetism, mechanics and thermodynamics is treated in a manner which is a direct generalization of Coleman and Noll's method. Maxwell stress tensors, ponderomotive forces, equations of motion and of energy and many other results all follow by a process of mathematical, and therefore logical, deduction.

From a strictly logical point of view, Chapters 14 and 15 should precede Chapter 8. But this would entail a serious drawback: students become impatient when they are presented with all the principles and theoretical deductions before encountering any real application.

The results of Chapter 15 are applied in Chapter 16, on magnetohydrodynamics, and in Chapters 17 and 18, which deal with electric and magnetic properties of materials, such as piezoelectricity, ferroelectricity, ferromagnetism and superconductivity.

To some degree, the choice of subjects invariably reflects a personal preference. I have tried to convince the reader that electromagnetism is a theory, not only of phenomena, but also of materials, and that it can be readily applied to moving, as well as to stationary, bodies. I have also attempted to dispel some common prejudices (such as the belief, which most students share, that electromagnetism is a linear theory) and misconceptions (for example, that the theory needs to be supplemented by a principle of 'causality').

Appendix A introduces the Gaussian system of electromagnetic units and deals with the relations (conversion, or rather correspondence, factors) between Gaussian and SI units.

Fifty solved example problems are scattered throughout the book. In addition, there are 175 exercises. Some of these are really parts of a mathematical proof or derivation that have been left to the reader. The remaining exercises, representing applications of the theory, are of the kind usually found at the end of a textbook chapter. Appendix B provides detailed solutions of all the exercises, but students are urged to try and solve the exercises before looking at these solutions.

The prerequisites for a student who intends to use this book as a text for an intermediate or advanced course on electromagnetic theory should present no problem. The mathematical tools required are no more than vector calculus and Legendre polynomials. Tensors are introduced for the purpose of formulating the basic laws of electromagnetism in the most general manner, and for dealing efficiently with changes of frame. But a prior familiarity with tensor calculus is not required; and beyond the first five chapters tensors are hardly ever mentioned. Of the physics curriculum, besides an introductory course on electromagnetism, some familiarity with the basic concepts of analytical mechanics, special relativity and thermodynamics is assumed, but any of these can be studied concurrently with a reading of this book.

I would like to thank Leon Mestel for a critical reading of the manuscript, and for many helpful comments. I am very grateful to my wife Dina Prialnik for numerous

comments and suggestions, and for her constant encouragement during the writing of this book. Finally, it gives me special pleasure to thank my daughter Michal Semo, Head of Tel Aviv University's Graphic Design Office, for all the graphical work.

*Tel Aviv 1999*

A. K.

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